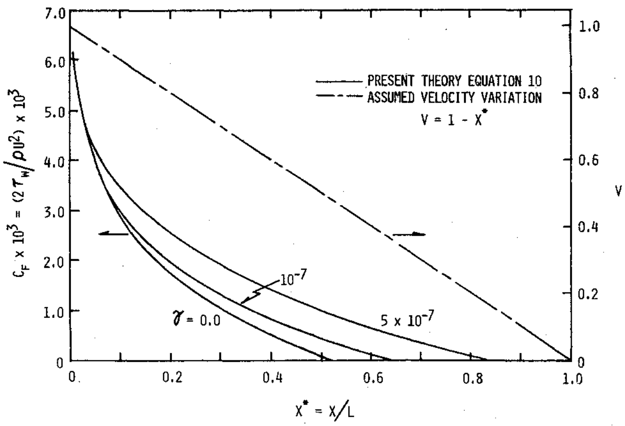

Fig. 2 Skin friction results – adverse pressure gradient flow.

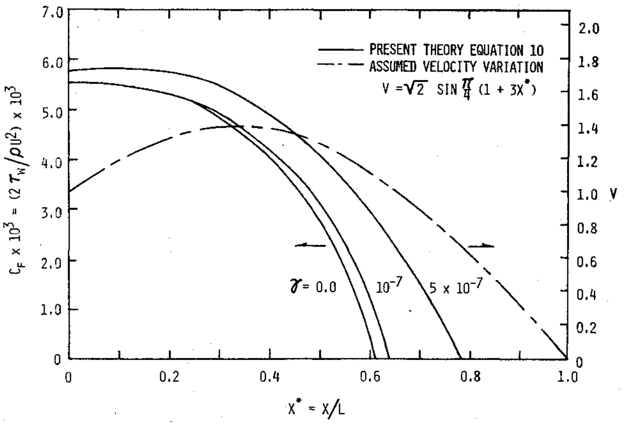

Fig. 3 Skin friction results – a favorable followed by strongly adverse pressure gradient flow.

The dependent variables u^+ and ψ^+ are both functions of y^+, α, and β which are functions of x. After calculating all streamwise gradients by use of the chain rule for partial derivatives, Eq. (9) may be integrated across the boundary layer. After some algebraic manipulations this yields

$$\frac{d\lambda}{dx^*} = RV(1-H)/G + V'\lambda/V \qquad (10)$$

in which the following quantities have been defined for convenience: $x^* = x/L$, $V = U/U_o$, $R = U_oL/\nu$, and $\lambda = V(2/C_f)^{1/2}$ where U_o and L are constant reference quantities.

Equation (10) is a first-order ordinary differential equation for skin friction as a function of x^*. The quantities G and H are integral coefficients defined by quadratures over the assumed velocity profile, Eq. (8). This equation contains a built-in separation criteria, in that if G goes to zero, then $d\lambda/dx^*$ goes to infinity, and C_f goes to zero. All that is needed to solve Eq. (10) is an initial value of C_f and knowledge of the outer flow variations. For further details of this derivation see Ref. 4.

Two previous studies similar to the present analysis were carried out by Napolitano,[2] and by Kruger and Sonju.[5] The former considered an isobaric freestream, while the latter considered a constant freestream velocity. The results obtained by application of Eq. (10) to those problems compare qualitatively with the earlier results, being approximately 15% lower in the first instance and 20% higher in the second. Figure 1 shows a comparison of the velocity profiles used in these early studies with Eq. (8).

Since the present analysis includes pressure gradient effects, several sample calculations for different pressure gradient flows, at different values of the interaction parameter $\gamma = \sigma B_o^2 \nu / \rho U_o^2$, are presented on Figs. 2-4. Figure 2 shows the results for an adverse pressure gradient flow, Fig. 3 is for a flow which initially experiences a favorable pressure gradient before becoming adverse, and Fig. 4 presents results for a relaxing flow. These plots show skin friction as a function of x^*, and also show the assumed variation of the freestream velocity. In all cases the skin friction is higher with the magnetic fields present than for the zero interaction case. In the adverse pressure gradient cases the presence of the magnetic fields are assumed to be applied only near the separation point, rather than over the entire surface.

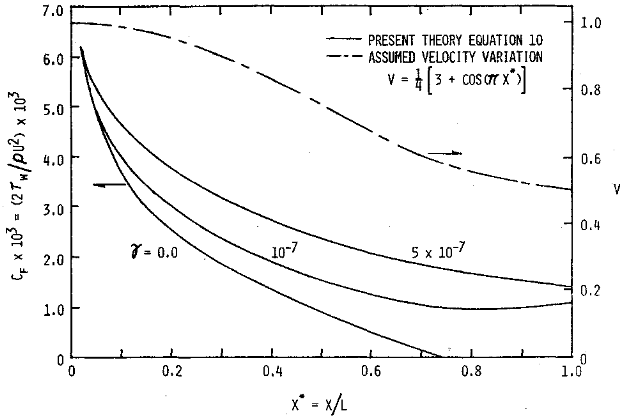

Fig. 4 Skin friction results – relaxing flow.

For the results presented in this paper, the interaction parameter γ and the length Reynolds number R are of the order of magnitude 10^{-7} and 10^7, respectively. The product of the interaction parameter and the Reynolds number represents the ratio of the magnetic pressure to the dynamic pressure. For the magnetic fields to have a significant effect the product γR must be of order one. Therefore, for flow situations where the length Reynolds number becomes very large, only weak magnetic fields are necessary to alter the pressure distribution and significantly change the skin friction.

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Simplified Modeling of Multiquantum Deactivation Reactions

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Introduction

COMPUTER modeling of pulsed and cw chemical lasers generally involves a large number of reactions,[1-3] since

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the different vibrational levels of the excited molecule are treated separately. Serious computer time and core storage problems occur, especially when multiquantum deactivation[4] is included. This type of deactivation process replaces the one-step-at-a-time reaction with a larger number that increases linearly with v, where v denotes the vibrational level of the excited molecule. In this note, a technique is introduced which is based on reasonable assumptions, and which contracts these reactions into a smaller number, thereby making their inclusion practical for computer modeling.

Analysis

For convenience, consider a model that treats the first $m+1$ vibrational levels of HF(v). There are then $m(m+1)/2$ multiquantum reactions with deactivator M:

$$\mathrm{HF}(v)+M \underset{k^{v,v'}}{\overset{k_{v,v'}}{\rightleftharpoons}} \mathrm{HF}(v')+M$$

where $1 \leq v \leq m, 0 \leq v' \leq v-1$. Assume that the backward rate coefficient $k^{v,v'}$ is zero, since the exponential factor in the equilibrium expression

$$k^{v,v'}=k_{v,v'}\exp(-\Delta E/RT)$$

is negligibly small for temperatures T of interest. The kinetic rate equations for the concentrations HF(v) then can be written as

$$\frac{d\mathrm{HF}(0)}{dt}=M\sum_{v'=1}^{m} k_{v',0}\mathrm{HF}(v')$$

$$\vdots$$

$$\frac{d\mathrm{HF}(v)}{dt}=M\sum_{v'=v+1}^{m} k_{v',v}\mathrm{HF}(v')-M\cdot\mathrm{HF}(v)\sum_{v'=0}^{v-1} k_{v,v'}$$

$$\vdots$$

$$\frac{d\mathrm{HF}(m)}{dt}=-M\cdot\mathrm{HF}(m)\sum_{v'=0}^{m-1} k_{m,v'} \qquad (1)$$

Next assume that all transitions from a given level v are equally probable, i.e., $k_{v,v'}=k_v$. Additionally, the assumption is made that $a_{v,v'} \equiv (k_{v'}/k_v), v \leq v' \leq m$, is independent of T, and thus is constant. Both assumptions are in good accord with the multiquantum deactivation rates calculated by Wilkins.[4] The rate coefficients and the two summations in Eq. (1) become

$$k_{v',v}=k_{v'}=a_{1,v'}k_1 \qquad k_{v,v'}=k_v=a_{1,v}k_1$$

$$\sum_{v'=0}^{v-1} k_{v,v'}=va_{1,v}k_1 \qquad \sum_{v'=v+1}^{m} k_{v',v}\mathrm{HF}(v')=k_1M_v$$

$$M_v \equiv \sum_{v'=v+1}^{m} a_{1,v'}\mathrm{HF}(v') \qquad v=0,\ldots,m-1 \qquad (2)$$

The rate equations now have the form

$$d\mathrm{HF}(0)/dt=k_1MM_0$$

$$\vdots$$

$$d\mathrm{HF}(v)/dt=k_1MM_v-(va_{1,v}k_1)M\cdot\mathrm{HF}(v)$$

$$\vdots$$

$$d\mathrm{HF}(m)/dt=-(ma_{1,m}k_1)M\cdot\mathrm{HF}(m)$$

These equations can be constructed directly by nonequilibrium computer codes from the $m+1$ "reactions"

$$\mathrm{HF}(v)+M \underset{k'_{-v}}{\overset{k'_v}{\rightleftharpoons}} M_v+M \qquad v=0,\ldots,m$$

where $k'_v=va_{1,v}k_1$, $k'_{-v}=k_1$, and M_v is given by Eq. (2).

The number of reactions for modeling the multiquantum process thus has been reduced from $m(m+1)/2$ to $m+1$. For HF chain reaction modeling, where m is at least 7, the reduction is from 28 to 8 reactions per process. Furthermore, the number of deactivation processes thought to be multiquantum is steadily increasing. Note that both k'_v and k'_{-v} have the usual Arrhenius temperature-dependent form. Both coefficients are required, and their ratio is not an equilibrium constant. Since the $a_{1,v}$ are constant, the M_v, Eq. (2), have a simple form commonly used in computer codes for collisional deactivators. The new "reactions" do not conserve atoms in the usual fashion. Nevertheless, this formulation is useful, since few, if any, computer programming changes are required for its implementation.

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Simple Waves and the Transonic Similarity Parameter

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Introduction

THE transonic small perturbation equation for the perturbed potential (in two dimensions) has the form

$$\varphi_{yy}=\beta^2\varphi_{xx}+\Gamma(M)\varphi_x\varphi_{xx} \qquad (1)$$

where $\beta^2=M^2-1$, and the potential φ has been normalized with respect to the upstream velocity. In the original derivation of Eq. (1) by von Karman,[1] $\Gamma=\gamma+1$, and in the course of time various other suggestions for $\Gamma(M)$ have been made. Spreiter[2] pointed out that this coefficient is ambiguous [to $0(\beta^2)$] and went on to suggest, $\Gamma_s=(\gamma+1)M^2$, which he demonstrated to be effective in transonic scaling. Hayes,[3] also noting this ambiguity, showed three such naturally occurring coefficients, and particularly noteworthy for our discussion is $\Gamma_H=(\gamma+1)M^4$, which appeared in an earlier paper, Hayes.[4] More recently Murman and Krupp[5] (see also

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